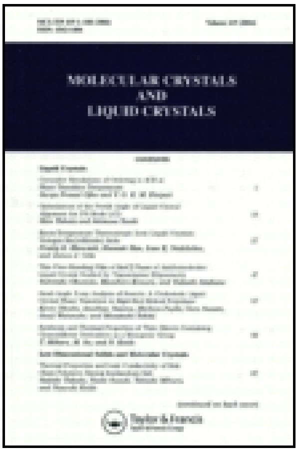

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Stokes parameters are derived for an interference pattern when superimpose light beams that propagate at a small angle with respect to each other. Our calculations allow us to separate the information about the intensity, the absolute phase and the polarization by means of correlation functions, a correlation function for intensity, one for the absolute phase and four for the polarization (one for each Stokes parameter).

1. Introduction

The concept of optical coherence (see [1] and its citations for the story of these studies) is associated with interference, because it is the simplest phenomenon that reveals correlation between light beams and Young's interference experiment has played a pivotal role in the development of optic and quantum physic [2]. Initially, the analyses have been performed almost exclusively in scalar description. From the first studies on the effect of the polarization state on the phenomena of interference [3] there were many studies on the interference of polarized light, see [4–11] just to list few schemes. It has been known for quite long time that though two coherent but orthogonally polarized beams do not give rise to intensity-interference fringes, they lead to a spatially varying polarization pattern, that is very suitable for the fabrication of holographic grating in polarization-sensitive materials [12–15]. A mathematical description for Young double slit experiment with polarized light was presented characterizing the resulting field by the Stokes parameters SP [16]. For a suitable exposition of coherency matrix theory and of the SP see [17]. Despite of long history of this problem, it has again become urgent in last years and has attracted the attention of many researchers (see for example [18–28]).

The Jones vectors of quasi-monochromatic light beam at a fixed point in space and at a certain fixed time can be represented as

$$\mathbf{E}_{\mathrm{i}} = \begin{pmatrix} \mathrm{A}_{\mathrm{x}}^{(\mathrm{i})} \, e^{i\delta_{\mathrm{x}}^{(\mathrm{i})}} \\ \mathrm{A}_{\mathrm{y}}^{(\mathrm{i})} \, e^{i\delta_{\mathrm{y}}^{(\mathrm{i})}} \end{pmatrix} \tag{1}$$

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where $A_x^{(i)}$ and $A_y^{(i)}$ are the amplitudes, $\delta_x^{(i)}$ and $\delta_y^{(i)}$ are the phases and "i" indicates the i-th beam of N beams (Jones vectors are explained in [17], the meaning of x and y is obvious). For brevity, we omit on the time and space dependence of various quantities, but they are assumed to vary with time unless we don't specify the contrary, i.e. the intensities, the polarization, etc. may fluctuate with time for example in random manner. The SP of the electric fields expressed in (1) are

$$\begin{aligned} S_0^{(i)} &= I_i; & S_1^{(i)} &= I_i \cos(2\theta_i); \\ S_2^{(i)} &= I_i \sin(2\theta_i)\cos\delta^{(i)}; & S_3^{(i)} &= I_i \sin(2\theta_i)\sin\delta^{(i)} \end{aligned} \tag{2}$$

where $I_i = \sqrt{(A_x^{(i)})^2 + (A_y^{(i)})^2}$ is the intensity, $\theta_i = \text{arctn}(A_y^{(i)}/A_x^{(i)})$ is the azimuth and $\delta^{(i)} = \delta_y^{(i)} - \delta_x^{(i)}$ is the relative phase. Moreover we introduce the absolute phase $\delta_m^{(i)} = (\delta_x^{(i)} + \delta_y^{(i)})/2$. Recently in [29] the superposition of N quasi-monochromatic coaxial light beams has been studied. The SP of the resulting beam S_a^{TOT} (the superposition of the beams) have been calculated in a given time instant and in a certain space position

$$S_a^{TOT} = \sum_{i=1}^{N} S_a^{(i)} + \sum_{(i,j)} \sqrt{I_i I_j}\,\mathbf{P}_a^{(i,j)} \cdot \mathbf{M}^{(i,j)} \tag{3}$$

where a = 0,1,2,3 and (i,j) indicates all the pairs of SP with i > j. In equations (3) the information on polarization, absolute phase and intensity is separated by introducing the vectors $\mathbf{P}_0^{(i,j)}$, $\mathbf{P}_1^{(i,j)}$, $\mathbf{P}_2^{(i,j)}$ and $\mathbf{P}_3^{(i,j)}$, which contain information only on the polarization, and the vector $\mathbf{M}^{(i,j)}$, that gives information on absolute phases. Now the information on intensities is only a factor, which is shared by all the interference terms. You can find the definition of these vectors in [29], but for completeness we report their values

$$\begin{aligned} \mathbf{P}_0^{(i,j)} &= \left(\cos\left(\theta_j - \theta_i\right)\cos\left(\frac{\delta^{(j)} - \delta^{(i)}}{2}\right),\quad \cos\left(\theta_j + \theta_i\right)\ \sin\left(\frac{\delta^{(j)} - \delta^{(i)}}{2}\right)\right); \\ \mathbf{P}_1^{(i,j)} &= \left(\cos\left(\theta_j + \theta_i\right)\ \cos\left(\frac{\delta^{(j)} - \delta^{(i)}}{2}\right),\quad \cos\left(\theta_j - \theta_i\right)\ \sin\left(\frac{\delta^{(j)} - \delta^{(i)}}{2}\right)\right); \\ \mathbf{P}_2^{(i,j)} &= \left(\sin\left(\theta_j + \theta_i\right)\ \cos\left(\frac{\delta^{(j)} + \delta^{(i)}}{2}\right),\quad \sin\left(\theta_i - \theta_j\right)\ \sin\left(\frac{\delta^{(j)} + \delta^{(i)}}{2}\right)\right); \\ \mathbf{P}_3^{(i,j)} &= \left(\sin\left(\theta_j + \theta_i\right)\ \sin\left(\frac{\delta^{(j)} + \delta^{(i)}}{2}\right),\quad \sin\left(\theta_j - \theta_i\right)\ \cos\left(\frac{\delta^{(j)} + \delta^{(i)}}{2}\right)\right); \\ \mathbf{M}^{(i,j)} &= \left(\cos\left(\delta_m^{(j)} - \delta_m^{(i)}\right), \sin\left(\delta_m^{(j)} - \delta_m^{(i)}\right)\right). \end{aligned} \tag{4}$$

Generally a measurement of SP is an average in the time, then in [29] the time-average of equations (3) was been calculated

$$\left\langle S_a^{TOT}\right\rangle = \sum_{i=1}^{N}\left\langle S_i^{(i)}\right\rangle + 2\sum_{(i,j)}\left\langle\sqrt{I_i I_j}\right\rangle\left|\left\langle\mathbf{P}_i^{(i,j)}\right\rangle\right|\left|\left\langle\mathbf{M}^{(i,j)}\right\rangle\right| \cos\overline{\delta}_a^{ij} \tag{5}$$

with a = 0,1,2,3 and where $\overline{\delta}_i^{12}$ is the angle between $\langle \mathbf{P}_i^{(1,2)} \rangle$ and $\langle \mathbf{M}^{(1,2)} \rangle$. We denote the time-average of the function f(t) by $\langle f(t) \rangle$, i.e.

$$\langle f(t) \rangle = \frac{1}{T} \int_0^T f(t)\, dt \tag{6}$$

where T is an interval of time long enough to make the time-average integral independent of T itself. The physical interpretation of these quantities is of correlations. In fact the module of vectors $|\langle \mathbf{P}_a^{(1,2)} \rangle|$, with a = 0,1,2,3, is polarization correlation functions. We have a polarization correlation function for each SP (the correlation function of the first Stokes parameter first was introduced in [30] in order understand a depolarization effect caused by non-uniform polarization distribution over the beam cross section which was discussed [31–33]). These correlations depend only from polarizations of the superposed beams. The function $2\langle \sqrt{I_1 I_2} \rangle$ normalized with $\langle I_1 + I_2 \rangle$ is the intensity correlation function, and $|\langle \mathbf{M}^{(1,2)} \rangle|$ is the absolute phase correlation function. We note $|\langle \mathbf{M}^{(1,2)} \rangle|$ is the classical correlation function in the scalar theory [1].

In this paper we study the polarization proprieties of a pattern formed when light beams with small angle respect the normal at the incident plane are superposed. Using small angles the z-component of the resulting field can be neglected (see [27,28] where the problem of z-component is treated).

2. Theoretical Approach

N beams of quasi-monochromatic light intersect with a small angles, such that $\sin\theta \sim \theta$. We consider the general case when these beams are partially polarized and, moreover, the intensity and the absolute phase also vary with time. We suppose $\mathbf{E}_i$ is the Jones vector of the i-th electric field along the axis z (see equation (1)). Then we can obtain the inclined beam field rotating $\mathbf{E}_i$ of an angle α_i around the x axis and of an angle β_i around the y axis. The two rotation matrices are respectively

$$R(\alpha_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i & \sin\alpha_i \\ 0 & -\sin\alpha_i & \cos\alpha_i \end{pmatrix}, \quad R(\beta_i) = \begin{pmatrix} \cos\beta_i & 0 & \sin\beta_i \\ 0 & 1 & 0 \\ -\sin\beta_i & 0 & \cos\beta_i \end{pmatrix}, \tag{7}$$

The inclined electric filed results from the multiplication of these two matrices and $\mathbf{E}_i$. If α_i and β_i are small the multiplication of these matrices is commutative and we obtain

$$R(\alpha_i, \beta_i) \cong \begin{pmatrix} 1 & 0 & \beta_i \\ 0 & 1 & \alpha_i \\ -\beta_i & -\alpha_i & 1 \end{pmatrix}. \tag{8}$$

Moreover it is easy to show that $R(\alpha_i, \beta_i)\, \mathbf{E}_i \cong \mathbf{E}_i$, i.e. the z-component of the electric field is negligible. In Fig. 1 we show the generic field $\mathbf{E}$ that is rotate by the matrix $R(\alpha, \beta)$. We note the light wavefront (that has the same absolute phase) comes on the screen after covering a distance which depends on the point of the screen, and then the absolute phase on the screen depends on x and y coordinates.

For simplicity we first analyze the case N = 2, i.e. only two fields $\mathbf{E}_1$ and $\mathbf{E}_2$ are superposed. We rotate the first one using the matrix $R(\alpha_1, \beta_1)$ and the second one with $R(\alpha_2, \beta_2)$. The rotations give a contribute only in absolute phase and then the superposition of the two beam gives

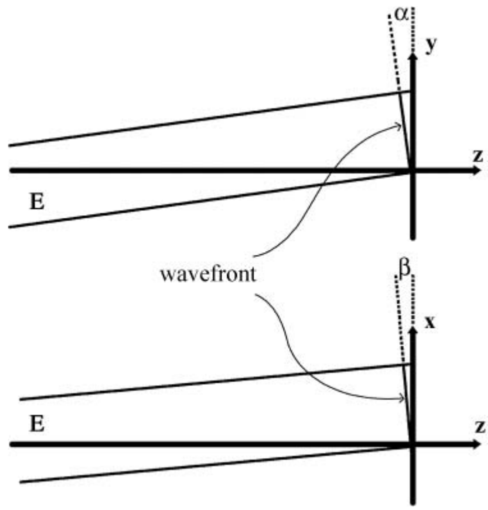

Figure 1. The generic field **E** along z axis that is rotated by the matrix R(α, β) and that impinges on a screen in x–y plane.

$$\mathbf{E}(\mathrm{x,\,y}) = \mathbf{E}_1 e^{\mathrm{i}k(\alpha_1\,x\,+\,\beta_1\,y)} + \mathbf{E}_2 e^{\mathrm{i}k(\alpha_2\,x\,+\,\beta_1\,y)} \tag{9}$$

Introducing the absolute phase $\delta_m^{(i)} = (\delta_x^{(i)} + \delta_y^{(i)})/2$ and the relative phase $\delta^{(i)} = \delta_y^{(i)} - \delta_x^{(i)}$ (in this case i = 1,2), we can rewrite equations (8) and we have

$$\mathbf{E}(\mathrm{x,\,y}) = \sqrt{I_1}\begin{pmatrix} \cos\theta_1\, e^{-i\delta^{(1)}/2} \\ \sin\theta_1\, e^{+\,i\delta^{(1)}/2} \end{pmatrix} e^{i\left(\delta_m^{(1)}\,+\,k(\alpha_1\,x\,+\,\beta_1\,y)\right)}$$

$$+\sqrt{I_2}\begin{pmatrix} \cos\theta_2\, e^{-i\delta^{(2)}/2} \\ \sin\theta_2\, e^{+\,i\delta^{(2)}/2} \end{pmatrix} e^{i\left(\delta_m^{(1)}+k(\alpha_2\,x+\beta_2\,y)\right)}. \tag{10}$$

We calculate the SP of **E**(x, y) confronting equation (10) with equation

$$\mathbf{E}_{TOT} = \sqrt{I_1}\begin{pmatrix} \cos\theta_1\, e^{-i\delta^{(1)}/2} \\ \sin\theta_1\, e^{+i\delta^{(1)}/2} \end{pmatrix} e^{i\delta_m^{(1)}} + \sqrt{I_2}\begin{pmatrix} \cos\theta_2\, e^{-i\delta^{(2)}/2} \\ \sin\theta_2\, e^{+i\delta^{(2)}/2} \end{pmatrix} e^{i\delta_m^{(2)}} \tag{11}$$

obtained in [29] for the overlapping of two coaxial light beams. We note that we can obtain equation (10) from equation (11) making the substations $\delta_m^{(i)} \to \delta_m^{(i)} + k(\alpha_i\,x + \beta_i\,y)$, for i = 1,2. Then the Stokes parameters of the superposition of the light beams are given from equation (3) in the case N = 2

$$S_a^{TOT}(\mathrm{x,\,y}) = S_a^{(1)} + S_a^{(1)} + 2\sqrt{I_1 I_2}\,\mathbf{P}_a^{(1,2)} \cdot \mathbf{M}^{(1,2)} \tag{12}$$

where $\mathbf{P}_a^{(1,2)}$ does not change respect equations (4), while

$$\mathbf{M}^{(1,2)} = \Big(\cos\left(\delta_m^{(2)} - \delta_m^{(1)} + k[(\alpha_2 - \alpha_1)\,x + (\beta_2 - \beta_1)\,y]\right),$$

$$\sin\left(\delta_m^{(2)} - \delta_m^{(1)} + k[(\alpha_2 - \alpha_1)\,x + (\beta_2 - \beta_1)\,y]\right)\Big). \tag{13}$$

The time average of equations (12) are

$$\left\langle S_a^{TOT}(x,y)\right\rangle = \left\langle S_a^{(1)}\right\rangle + \left\langle S_a^{(2)}\right\rangle + 2\left\langle\sqrt{I_1 I_2}\right\rangle\left\langle \mathbf{P}_a^{(1,2)}\right\rangle \cdot \left\langle \mathbf{M}^{(1,2)}\right\rangle, \tag{14}$$

where the average of a generic vector $\mathbf{V}$ mean $\langle \mathbf{V}\rangle = \left(\left\langle V_1\right\rangle,\ \left\langle V_2\right\rangle\right)$ and still $a = 0,1,2,3$. Equation (12) can be rewrite in this way

$$\left\langle S_a^{TOT}(x,y)\right\rangle = \left\langle S_a^{(1)}\right\rangle + 2\left\langle\sqrt{I_1 I_2}\right\rangle\left|\left\langle \mathbf{P}_a^{(1,2)}\right\rangle\right|\ \left|\left\langle \mathbf{M}^{(1,2)}\right\rangle\right| \times \cos\left(\bar{\delta}_a^{12} + k[(\alpha_2 - \alpha_1)\,x + (\beta_2 - \beta_1)\,y]\right), \tag{15}$$

where $\bar{\delta}_i^{12}$ is the angle between $\langle \mathbf{P}_i^{(1,2)}\rangle$ and $\langle \mathbf{M}^{(1,2)}\rangle$ in the origin of the Cartesian axes. The meaning of each factor in the interference term is clear, $2\langle\sqrt{I_1 I_2}\rangle$ normalized with $\langle I_1 + I_2\rangle$ is the correlation between intensities, which depends from the intensity fluctuation of the two electric fields. $|\langle \mathbf{P}_a^{(1,2)}\rangle|$ is the correlation between the state of polarization of the a-th SP, which does not change respect the case discussed in [29]. $|\langle \mathbf{M}^{(1,2)}\rangle|$ is the absolute phase correlation and does not depend on point x and y.

If we have N the electric fields $\mathbf{E}_i$ we can generalize equations (15) and we obtain

$$\left\langle S_a^{TOT}(x,y)\right\rangle = \sum_{i=1}^{N}\left\langle S_a^{(i)}\right\rangle + \sum_{(i,j)} 2\left\langle\sqrt{I_1 I_2}\right\rangle\left|\left\langle \mathbf{P}_a^{(i,j)}\right\rangle\right|\ \left|\left\langle \mathbf{M}^{(i,j)}\right\rangle\right| \times \cos\left(\bar{\delta}_a^{ij} + k[(\alpha_j - \alpha_i)\,x + (\beta_j - \beta_i)\,y]\right) \tag{16}$$

for $a = 0,1,2,3$ and with

$$\mathbf{M}^{(i,j)} = (\cos(\delta_m^{(j)} - \delta_m^{(i)} + k[(\alpha_j - \alpha_i)\,x + (\beta_j - \beta_i)\,y]), \sin(\delta_m^{(j)} - \delta_m^{(i)} + k[(\alpha_j - \alpha_i)\,x + (\beta_j - \beta_i)\,y])). \tag{17}$$

In the case the pattern is more complicate and it is not a simple modulation of the SP.

3. Conclusion

In this paper we studied the polarization modulation in light pattern when we superpose quasi-monochromatic light beams with small angle respect the normal at the incident plane. The intensities, the polarization and the absolute phase of these fields can fluctuate in time. We calculate the SP of the superposition of the beams in a point (x,y) of screen (see figure 1). In our work we are able to separate information on intensity, polarization and absolute phase by the vectors $\mathbf{P}_0^{(1,2)}$, $\mathbf{P}_1^{(1,2)}$, $\mathbf{P}_2^{(1,2)}$ and $\mathbf{P}_3^{(1,2)}$, which contain information only on the polarization, and the vector $\mathbf{M}^{(1,2)}$, that gives information on absolute phases. These vectors are introduced initially in [29] for the superposition of coaxial electro-magnetic beams.

We note the difference respect the case analyzed in [29] is that $\mathbf{M}^{(1,2)}$ depends on the point (x,y) i.e. the vector $\mathbf{M}^{(1,2)}$ rotates in different position of the screen and then intensity and polarization pattern is formed. In equations (15) and (16) we calculated the time average the instantaneous SP. In this way we can observe the amplitudes of the modulation (the contrast) of each SP are the product of three correlation functions, i.e. the intensity correlation function $2\langle\sqrt{I_1 I_2}\rangle/\langle I_1 + I_2\rangle$, the absolute phase correlation function $|\langle \mathbf{M}^{(i,j)}\rangle|$ and the relative polarization correlation functions $|\langle \mathbf{P}_a^{(i,j)}\rangle|$.

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